

Stationary states and screening equations in the spin-Hall effect

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Abstract

The characterization of the stationary states in the spin-Hall effect is discussed within the framework of the phenomenological two spin-channel model. It is shown that two different definitions of the stationary states can be applied in the spin-Hall effect, leading to two different types of state in the bulk: zero transverse spin-current or non-zero pure spin-current. This difference is due to the treatment of the region near the edges, in which electric charge accumulation occurs. The screening equations that describe the accumulation of electric charges due to spin-orbit coupling are derived. The spin-accumulation associated to spin-flip scattering and the spin-Hall accumulation due to spin-orbit coupling are two independent effects if we assume that the screening length is small with respect to the spin-diffusion length. The corresponding transport equations are discussed in terms of the Dyakonov-Perel equations.

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I. INTRODUCTION

The spin-Hall effect (SHE) is one of the most important effect in the new field of spin-orbitronics. However, in spite of a large number of excellent reports about SHE (see [1–9] and references therein), a fundamental ambiguity seems to persist in the definition of the stationary regime, leading to different predictions as to the presence of pure spin-current on the stationary state [10].

The SHE describes the spin-accumulation generated at the edges of a paramagnet with strong spin-orbit coupling (SOC), while injecting an electric current [11]. The phenomenological description of the SHE is based on the Dyakonov-Perel transport equations [1, 2], which are a generalization of Ohm’s law for the spin-dependent electric carriers in the presence of SOC. As shown in Fig.1, the two spin-channel model can be used in order to describe the SHE: it suffices to assume that the spin-orbit scattering plays the same role as an effective magnetic field $\vec{H}_{so\uparrow}$ in a two-channel version of the Hall effect. The main property of this effective magnetic field is expressed by the reciprocity relation $\vec{H}_{so\uparrow} = \pm\vec{H}_{so}$: the effective magnetic field is acting as an external static magnetic field applied along the direction of the effective field $\vec{H}_{so\uparrow}$ for the electric carriers belonging to the \uparrow channel, while it is acting in the opposite direction for the electric carriers belonging to the \downarrow channel (see Fig.1). Consequently, the system can be viewed as a simple superimposition of two standard Hall devices with two populations of electric charges experiencing opposite magnetic fields. More precisely, the magnetic field along the $\vec{H}_{so\uparrow}$ direction generates a charge accumulation δn_{\uparrow} at the edge, while the magnetic field along the $\vec{H}_{so\downarrow}$ direction generates a charge accumulation $\delta n_{\downarrow} = -\delta n_{\uparrow}$ at the same edge. The superimposition of the two sub-systems leads to a vanishing electric charge accumulation $\delta n = \delta n_{\uparrow} + \delta n_{\downarrow} = 0$ and non-zero spin-accumulation $\Delta n = \delta n_{\uparrow} - \delta n_{\downarrow} \neq 0$.

The usual theoretical approach to the SHE consists in calculating the stationary state after summing over the total system, i.e. for zero charge accumulation [4–9]. The result is a generation of a transverse pure spin current $J_{y\uparrow} = -J_{y\downarrow} \neq 0$.

However, this generation of a pure spin current is not satisfying from the thermodynamical point of view. Indeed, the second law of thermodynamics imposes that the stationary state should correspond to the minimum power dissipation. As shown in a recent report based on this variational principle [10], the minimum power dissipation in the spin-Hall system

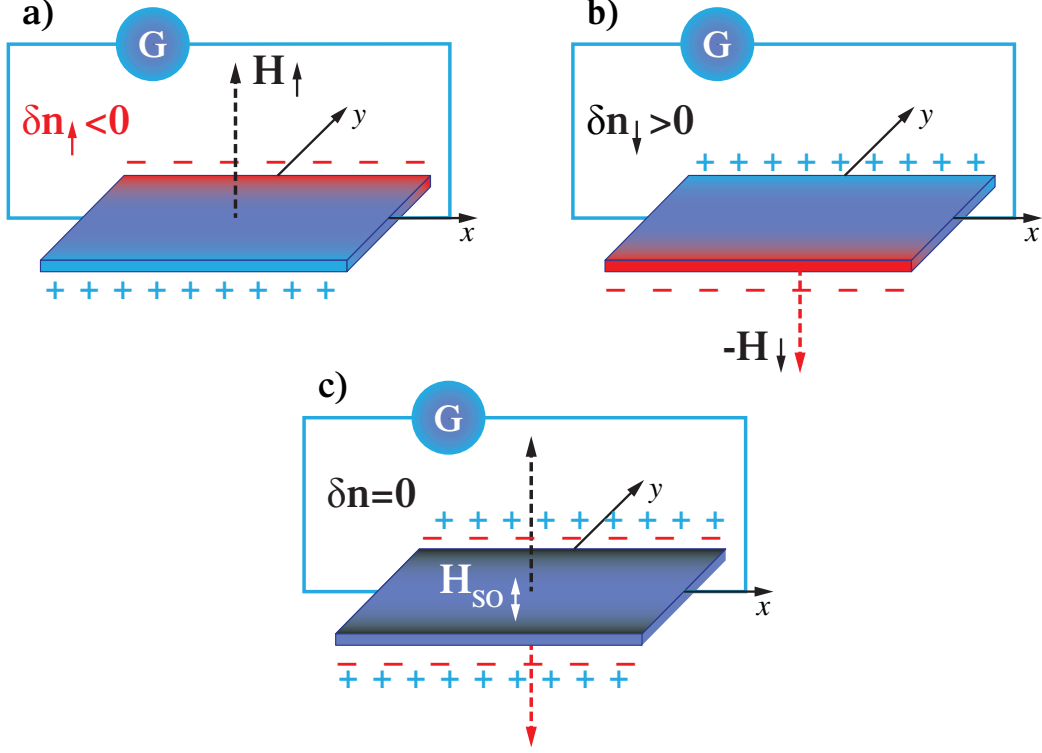


FIG. 1: : Schematic representation of the spin-Hall effect with the electrostatic charge accumulation δn_{\uparrow} at the boundaries: (a) usual Hall effect with a magnetic field \vec{H}_{\uparrow} in the up direction (b) usual Hall effect with a magnetic field \vec{H}_{\downarrow} in the down direction, (c) the addition of configuration (a) and (b) leads to the modelisation of the spin-orbit scattering assuming an effective magnetic field \vec{H}_{so} acting on the two different electric carriers, defining the spin up and spin down channels.

is reached for $J_{y\uparrow} = J_{y\downarrow} = 0$ in the bulk, except for Corbino configurations [10]. The corresponding system is then less constrained than the previous one.

The aim of this work is to study these two results in the framework of the usual definition $dn_{\uparrow}/dt = 0$ of the stationary state, and to understand the apparent contradiction on the basis of the screening equations. It is shown that the last result ($J_{y\uparrow} = J_{y\downarrow} = 0$) is obtained if the condition of stationary state is applied before summing over the two channels (with charge accumulation $\delta n_{\uparrow} \neq 0$) [3], while the former result ($J_{y\uparrow} = -J_{y\downarrow} \neq 0$) is obtained if the stationarity condition is apply after summing over the two channels (charge accumulation $\delta n = 0$).

The paper is organized as follow. The first section treats the standard Hall effect in the formalism of mesoscopic non-equilibrium thermodynamics. The screening equation is

derived and the usual approximations related to constant conductivity and small Debye-Fermi length are analyzed. The second section is devoted to the two spin-channel model for the SHE without spin-flip scattering, with the derivation of the screening equations. The third section presents the derivation of the screening equation that includes spin-flip scattering. The section four analyzes the corresponding transport equations in terms of the Dyakonov-Perel equations.

II. HALL EFFECT

The goal of this section is to characterize the stationary state of the standard Hall device on the basis of the law $dn/dt = 0$ (where n is the density of electric carriers), and investigating the approximations that consists in assuming constant conductivity and small screening length.

Ohm's law reads:

$$\vec{J} = -\hat{\eta}n\vec{\nabla}\mu, \quad (1)$$

where n is the density of electric carriers of charge q , μ is the chemical potential, and the mobility tensor $\hat{\eta}$ is related to the conductivity tensor $\hat{\sigma}$ by the relation $\hat{\eta} = \hat{\sigma}/(qn)$. If the sample under consideration is an isotropic planar layer, to which a magnetic field \vec{H} is applied along the direction \vec{e}_z perpendicular to the layer, the mobility tensor in the orthonormal basis $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ is expressed by the matrix:

$$\hat{\eta} = \begin{pmatrix} \eta & \eta_H \\ -\eta_H & \eta \end{pmatrix}, \quad (2)$$

where the coefficient η_H is the Hall mobility (which is an odd function of H). According to the Drude relation we have $\sigma = q^2n\tau/m^*$ (where τ is the relaxation time and m^* is the effective mass) so that $\eta = q\tau/m^*$ does not depend on n .

Introducing a unit vector \vec{p} in the direction of the applied field \vec{H} , the vector form of Eq.(1) reads:

$$\vec{J} = -\eta n\vec{\nabla}\mu + \vec{p} \times n\eta_H\vec{\nabla}\mu. \quad (3)$$

On the other hand, the chemical potential is defined by :

$$\mu = kT\ln(n/n_0) + V_G + V(\vec{H}) + \mu_0, \quad (4)$$

where T is either the Fermi temperature ($T = T_F$) in the case of a metal or the temperature of the thermostat in the case of a non-degenerated semi-conductor, k is the Boltzmann constant, n_0 is the constant density of the electric carriers that corresponds to the electric neutrality of the material, and μ_0 describes the chemical potential related to relaxation processes, which is also constant in the bulk material. The electric potential V_G is produced by the electric generator and imposes the constant electric field $qE_x^0 = -\partial V_G/\partial x$ along the x axis. On the other hand, the electric potential $V(\vec{H})$ takes into account the effect of the magnetic field \vec{H} . Due to the Lorentz force, the consequence of the application of the magnetic field is a redistribution of the electric carriers in the direction y , which results in a charge accumulation $\delta n = n - n_0$ at the edges. The charge accumulation generates, in turn, an electric field $qE_y = -\partial V(\vec{H})/\partial y$ defined by Gauss's law:

$$\frac{\partial E_y}{\partial y} = \frac{q\delta n}{\epsilon} \quad (5)$$

For convenience, the two contributions $\vec{\nabla}V_G$ and $\vec{\nabla}V(H)$ of the gradient of the chemical potential can be combined in a single electric field $q\vec{E} = -\vec{\nabla}V_G - \vec{\nabla}V(\vec{H}) = qE_x^0\vec{e}_x + qE_y\vec{e}_y$. Inserting the expression of the chemical potential Eq.(4) into Ohm's law Eq.(3) we obtain:

$$\vec{J} = qn\eta\vec{E} - D\vec{\nabla}n - \vec{p} \times (qn\eta\vec{E} - D_H\vec{\nabla}n), \quad (6)$$

where the diffusion constants are defined as $D = \eta kT$ and $D_H = \eta_H kT$.

The divergence of Eq.(6) reads:

$$\text{div}(\vec{J}) = qn\eta \text{div}(\vec{E}) + q\eta\vec{\nabla}n \cdot \vec{E} - D\nabla^2 n, \quad (7)$$

where we have used the vector relations $\text{div}(n\vec{E}) = \vec{E} \cdot \vec{\nabla}n + n \text{div}(\vec{E})$ and $\text{div}(\vec{p} \times \vec{\nabla}\mu) = \vec{\nabla}\mu \cdot \vec{rot}(\vec{p}) - \vec{p} \cdot \vec{rot}(\vec{\nabla}\mu) = 0$. The consequence of the last relation is that the Hall term in the right hand side of Eq.(6) disappears from the definition of the stationary states. Nevertheless, the Hall effect is still present as it is responsible for the charge accumulation δn , i.e. as it is responsible for the derivative of the electric field $\partial E_y/\partial y$. The stationarity condition writes $\partial n/\partial t = -\text{div}(\vec{J}) = 0$. In the case of a recombination between electrons and holes, or for other relaxation mechanisms, we should add the relaxation parameter \dot{R} such that $\text{div}(\vec{J}) = -\dot{R}$ (an explicit expression of \dot{R} can be found in [5]). The expression the stationarity condition leads to the following **screening equation** for the electric charge accumulation δn :

$$\frac{\partial^2 \delta n}{\partial y^2} - \frac{\delta n}{\lambda_D^2} = \frac{\partial \delta n}{\partial y} \frac{q^2}{\epsilon kT} \int_y \delta n(y') dy' + \frac{\dot{R}}{D} \quad (8)$$

where $\lambda_D = \sqrt{\frac{\epsilon k T}{q^2 n}}$ is the screening length. This is an exact result for the Hall bar assuming translation invariance along x . In the case $n \gg |\delta n|$, the screening length is the Debye length $\lambda_D \approx \bar{\lambda}_D \equiv \sqrt{\frac{\epsilon k T}{q^2 n_0}}$.

A. Constant conductivity approximation

When $|\delta n| \ll n_0$, the conductivity $\sigma = q\eta(n_0 + \delta n) \approx q\eta n_0$ is constant. If we further assume that $\dot{R} \approx 0$, then Eq.(8) reduces to the well-known screening equation at equilibrium:

$$\nabla^2 \delta n - \frac{\delta n}{\lambda_D^2} \approx 0. \quad (9)$$

The electrostatic charge accumulation decays exponentially to zero with a typical length $\bar{\lambda}_D$. Consequently, the last term $q\eta \vec{\nabla} n \cdot \vec{E}$ in the left hand side of Eq.(7) accounts for the variation of the conductivity σ due to the electrostatic charge accumulation over a distance λ_D from the border of the Hall device. The approximation $|\delta n| \ll n_0$ as been well established as correct for conventional systems.

B. Small Debye-Fermi length and bulk approximation

If we assume λ_D small, we have in second order:

$$-\delta n = \frac{\partial \delta n}{\partial y} \left(\frac{1}{n} \int_y \delta n(y') dy' \right) \approx \frac{\partial \delta n}{\partial y} \left(\frac{1}{n_0} \int_y \delta n(y') dy' \right), \quad (10)$$

A moderate non-conservation of the electric charges $\dot{R} \neq 0$ has no consequence in the framework of this approximation. The approximation of small Debye-Fermi length **B** together with that of constant conductivity **A** (the right-hand side of Eq.(10) is vanishing) results in the **bulk approximation** $\delta n \approx 0$.

III. SPIN-HALL EFFECT IN THE TWO SPIN CHANNEL MODEL

The goal of this section is to introduce the two spin channels labeled by the index \uparrow, \downarrow , and to characterize the stationary states defined by the conservation laws $dn_{\uparrow}/dt = -\text{div} \vec{J}_{\uparrow} - \dot{R} = 0$ (in the absence of spin-flip scattering).

The spin-dependent conductivity σ_{\uparrow} of the spin-channels can be introduced through the Drude formula: $\sigma_{\uparrow} = q^2 n_{\uparrow} \tau / m^*$ where the relaxation τ is not spin-dependent, and m^* is

the effective mass. Hence, the mobility $\eta = \sigma_{\uparrow}/(qn_{\uparrow})$ does not depend on n_{\uparrow} and is spin independent. Consequently, the diffusion constant is also spin-independent $D = \sigma_{\uparrow}kT/n_{\uparrow} = \eta kT$. In contrast, the spin-orbit diffusion is spin-dependent $D_{so\uparrow} = \eta_{so\uparrow}kT = \pm\eta_{so}kT$ (where the sign (+) corresponds to the spin \uparrow and the sign (−) corresponds to the spin \downarrow), with the relation $\eta_{so\uparrow} = -\eta_{so\downarrow} = \eta_{so}$. Ohm's law reads:

$$\vec{J}_{\uparrow} = -\hat{\eta}n_{\uparrow}\vec{\nabla}\mu_{\uparrow}, \quad (11)$$

where the mobility tensor is given by:

$$\hat{\eta}_{\uparrow} = \begin{pmatrix} \eta & \eta_{so} & 0 & 0 \\ -\eta_{so} & \eta & 0 & 0 \\ 0 & 0 & \eta & -\eta_{so} \\ 0 & 0 & \eta_{so} & \eta \end{pmatrix}. \quad (12)$$

In a vector form we have:

$$\vec{J}_{\uparrow} = -\eta n_{\uparrow}\vec{\nabla}\mu_{\uparrow} \pm \vec{p} \times n_{\uparrow}\eta_{so}\vec{\nabla}\mu_{\uparrow} \quad (13)$$

where the sign (+) corresponds to the spin \uparrow and the sign (−) corresponds to the spin \downarrow . As shown in the last Section, Eq.(13) is a generalization of the Dyakonov-Perel equations.

According to the model depicted in Fig.1, the chemical potential of the charge carriers in each spin channel is given by Eq.(4):

$$\mu_{\uparrow} = kT \ln(n_{\uparrow}/n_0) + V_G + V(H_{so\uparrow}) + \mu_{0\uparrow}, \quad (14)$$

where V_G is the contribution of the electric generator which imposes a constant electric field along the x direction: $qE_x^0 = -\partial V_G/\partial x$. The contribution $V(H_{so\uparrow})$ takes into account the effect of SOC through the effective magnetic field $H_{so\uparrow}$, and $\mu_{0\uparrow}$ accounts for the spin-dependent properties of the electric carriers.

As described in Fig.1 and in the introduction, the effect of the effective magnetic field $H_{so\uparrow} = \pm H_{so}$ is equivalent to that of the usual Hall effect for each spin-channel. It leads to a redistribution of the electric charges δn_{\uparrow} such that $\delta n_{\uparrow} = -\delta n_{\downarrow}$. This distribution defines a spin-dependent electric field $E_{y\uparrow}$ along the y -direction through Gauss's law $\partial E_{y\uparrow}/\partial y = q\delta n_{\uparrow}/\epsilon$. The two contributions V_G and $V(H_{so\uparrow})$ in Eq.(14) can be combined into a single electric field $q\vec{E}_{\uparrow} = -\vec{\nabla}V_{\uparrow} = -\vec{\nabla}V_G - \vec{\nabla}V(H_{so\uparrow}) = qE_x^0\vec{e}_x + qE_{y\uparrow}\vec{e}_y$. Inserting the expression

of the chemical potential Eq.(14) into the Omh's law Eq.(13) yields:

$$\vec{J} = qn_{\uparrow}\eta\vec{E}_{\uparrow} - D\vec{\nabla}n_{\uparrow} \mp \vec{p} \times (qn_{\uparrow}\eta\vec{E}_{\uparrow} - D_H\vec{\nabla}n_{\uparrow}), \quad (15)$$

It is important to note here that the choice of V_{\uparrow} instead of V (or \vec{E}_{\uparrow} instead of \vec{E}) in the expression of the chemical potential Eq.(14) corresponds to the model depicted in Fig.1 where each spin channel is treated separately. If the two sub-systems of Fig.1, are combined before applying the stationarity condition, the symmetry property $\delta n_{\uparrow} = -\delta n_{\downarrow}$ related to the two sub-systems reduces to a property of the total system $\delta n = \delta n_{\uparrow} + \delta n_{\downarrow} = 0$. The corresponding system is then analogous to the Corbino configuration [10] in which charge accumulation is forbidden (due to the absence of edge), and this configuration imposes a Hall current for both subsystems in the stationary state: we have $J_{y\uparrow} = -J_{y\downarrow}$, instead of $J_{y\uparrow} = J_{y\downarrow} = 0$. In agreement with the Dyakonov-Perel equations, most publications about the SHE [8] assumed implicitly the more constrained situation $\delta n = 0$ and the generation of a pure spin-current. The case of the less constrained system $\delta n_{\uparrow} = -\delta n_{\downarrow}$ is developed below, keeping in minde that the other situation is recovered with replacing \vec{E} instead of \vec{E}_{\uparrow} .

From Eq.(13), we derived the expression of the stationarity condition $div(\vec{J}_{\uparrow}) = -\dot{R}/2$:

$$-D\nabla^2 n_{\uparrow} + q\eta n_{\uparrow} div\vec{E}_{\uparrow} + q\eta\vec{\nabla}n_{\uparrow}.\vec{E}_{\uparrow} = -\dot{R}/2. \quad (16)$$

Equation (16) can be put into the form:

$$\frac{\partial^2 \delta n_{\uparrow}}{\partial y^2} - \frac{\delta n_{\uparrow}}{\lambda_{D\uparrow}^2} - \frac{q}{kT}\vec{\nabla}\delta n_{\uparrow}.\vec{E}_{\uparrow} = \frac{\dot{R}}{2D}, \quad (17)$$

where the screening length $\lambda_{D\uparrow} = \sqrt{\frac{\epsilon kT}{q^2 n_{\uparrow}}}$ is approximatively equal to the Debye length $\lambda_D \approx \sqrt{\frac{\epsilon kT}{q^2 n_0}}$.

Using $\partial E_{\uparrow}/\partial y = q\delta n_{\uparrow}/\epsilon$, equation (17) reduces to :

$$\lambda_{D\uparrow}^2 \left(\frac{\partial^2 \delta n_{\uparrow}}{\partial y^2} - \frac{\dot{R}}{2D} \right) - \delta n_{\uparrow} = \frac{\partial \delta n_{\uparrow}}{\partial y} \frac{1}{n_{\uparrow}} \int_y \delta n_{\uparrow} dy' \quad (18)$$

For each spin channel, Eq.(18) is equivalent to that of the simple Hall effect Eq.(8) because the two spin-dependent equations are decoupled. This is also equivalent to a simple screening equation of a transport process without Hall effect, in which the charge accumulation δn_{\uparrow} is imposed at the interfaces by other means.

In the approximation in which we assumed **constant conductivity** ($|\delta n| \ll n$) and $\dot{R} \approx 0$ we obtain:

$$\frac{\partial^2 \delta n_{\uparrow}}{\partial y^2} - \frac{\delta n_{\uparrow}}{\lambda_D^2} \approx 0, \quad (19)$$

The effect of the electrostatic charge accumulation decreases exponentially over the screening length λ_D : $\delta n_{\uparrow} \sim \delta n_{0\uparrow} e^{-y/\lambda_D}$ where $\delta n_{0\uparrow}$ is the charge accumulation at the edges produced by the effective field $\vec{H}_{so\uparrow}$. The spin accumulation takes place $\Delta n = \delta n_{\uparrow} - \delta n_{\downarrow} = 2\delta n_{\uparrow} \neq 0$ with zero total charge accumulation $\delta n = 0$ [2, 8].

On the other hand, in the approximation of small Debye length but without assuming constant conductivity, entails the relation:

$$-\delta n_{\uparrow} = \frac{\partial \delta n_{\uparrow}}{\partial y} \frac{1}{n_{\uparrow}} \int_y \delta n_{\uparrow} dy'. \quad (20)$$

The approximation of small Debye-Fermi length **B** together with that of constant conductivity **A** (the right-hand side of Eq.(10) is vanishing) results in the **bulk approximation** $\delta n_{\uparrow} \approx 0$ and $\Delta n \approx 0$. This result is also valid in the case of spin-independent field $\vec{E}_{\uparrow} = \vec{E}$ (or $V(H_{so\uparrow}) = V$ in Eq.(14)).

IV. SPIN-HALL EFFECT WITH SPIN-FLIP RELAXATION

It is easy to take into account phenomenologically the spin-flip relaxation in the framework of the two channel model. As a consequence, a well-known interface effect due to non-local giant magnetoresistance [13, 14] will be superimposed to the spin-Hall effect described above. Indeed, if the surface is replaced by an interface with a ferromagnetic material [4] the boundary conditions related to the spin system cannot be left free, and the situation described in the previous section should be revisited. In particular, it is necessary to take into account the gradient $\vec{\nabla} \mu_{0\uparrow}$ of the spin-dependent chemical potentials $\mu_{0\uparrow}$ near the interface. We must then consider four forces in the gradient of the chemical potential $\vec{\nabla} \mu_{\uparrow} = kT \vec{\nabla} \ln(n_{\uparrow}) - qE_x^0 \vec{e}_x - qE_{y\uparrow} \vec{e}_y + \vec{\nabla} \mu_{0\uparrow}$. The pumping force $\vec{\nabla}(\Delta \mu_0) \neq 0$ where $\Delta \mu_0 = \mu_{0\uparrow} - \mu_{0\downarrow}$ is responsible for the out-of-equilibrium state of the two spin populations [12]. This force also leads to the well-known giant magnetoresistance (GMR) effect [15–18].

In the framework of the two channel model, the condition for the spin conservation becomes:

$$\text{div}(\vec{J}_{\uparrow}) = -\frac{\dot{R}}{2} \mp L \Delta \mu_0 \quad (21)$$

where the spin-flip Onsager coefficient L is related to the spin diffusion length (see below) measured in GMR experiments [12, 15–18]. Note that L is inversely proportional the spin-relaxation time τ_{sf} . Equation (17) becomes:

$$\frac{\partial^2 \delta n_{\uparrow}}{\partial y^2} - \frac{\delta n_{\uparrow}}{\lambda_{D\uparrow}^2} - \frac{\partial \delta n_{\uparrow}}{\partial y} \frac{1}{\lambda_{D\uparrow}^2 n_{\uparrow}} \int_y \delta n_{\uparrow} dy' + \frac{1}{kT} \left(n_{\uparrow} \frac{\partial^2 \mu_{0\uparrow}}{\partial y^2} + \frac{\partial \mu_{0\uparrow}}{\partial y} \frac{\partial \delta n_{\uparrow}}{\partial y} \right) = \frac{1}{2\eta kT} \left(\dot{R} \pm 2L\Delta\mu_0 \right). \quad (22)$$

We will not try to solve this system of two coupled equations, but the analysis performed in the previous sections remains applicable.

Let us focus on the **bulk approximation** $\delta n_{\uparrow} \approx 0$, which is also valid in the region $\lambda_D \ll y \ll l_{sf}$, in which $\Delta\mu_0$ is not zero. Assuming also, for the sake of simplicity that $\dot{R} \approx 0$, Eq. (22) then reduces to:

$$\frac{\partial^2 \mu_{0\uparrow}}{\partial y^2} = \pm \frac{L}{n_0 \eta} \Delta\mu_0. \quad (23)$$

We recognize the spin-accumulation equation for $\Delta\mu_0$ which is formally equivalent to that of the spin-accumulation described in the context of GMR:

$$\frac{\partial^2 \Delta\mu_0}{\partial y^2} + \frac{\Delta\mu_0}{l_{sf}^2} = 0, \quad (24)$$

where $l_{sf} = \sqrt{\sigma_0/(qL)}$ with $\sigma_0 = \eta q n_0$. Independently, we can look at the approximation of small λ_D , without assuming constant conductivity. Multiplying Eq.(22) by λ_D^2 yields:

$$\lambda_D^2 \left(\frac{\partial^2 \delta n_{\uparrow}}{\partial y^2} - \frac{\dot{R}}{2D} \right) - \delta n_{\uparrow} - \frac{1}{n_{\uparrow}} \frac{\partial \delta n_{\uparrow}}{\partial y} \int_y \delta n_{\uparrow} dy' + \frac{\lambda_D^2}{kT} \left(n_{\uparrow} \frac{\partial^2 \mu_{0\uparrow}}{\partial y^2} + \frac{\partial \mu_{0\uparrow}}{\partial y} \frac{\partial \delta n_{\uparrow}}{\partial y} \right) = \pm \frac{\lambda_D^2}{l_{sf}^2} \Delta\mu, \quad (25)$$

In a metal, the Debye length λ_D is of the order of a nanometer while the spin-flip relaxation length l_{sf} is few tens of nanometers. To second order in λ_D , Eq.(25) reduces to:

$$-\delta n_{\uparrow} - \frac{1}{n_{\uparrow}} \frac{\partial \delta n_{\uparrow}}{\partial y} \int_y \delta n_{\uparrow} dy' \approx 0 \quad (26)$$

Equation Eq.(26) which describes the SHE with spin-flip scattering is the same as Eq.(20) which describe SHE without spin-flip scattering.

To conclude this section, we can state that despite the existence of the GMR-like spin-accumulation at the interface over a distance l_{sf} , there is no qualitative change introduced by the spin-flip relaxation in the bulk SHE, as long as the spin-diffusion length is much

larger than the Debye length $l_{sf} \gg \lambda_D$. In particular, the stationary state defined by $J_{y\uparrow} = 0$ is maintained in the bulk together with the spin-accumulation at the edges due to SOC. This SOC spin-accumulation is superimposed on the spin-accumulation related to the spin-flip and giant magnetoresistance described by Eq.(24). Furthermore, as in the previous section, this state is also compatible with the stationary state defined by the pure spin-current $J_{y\uparrow} = -J_{y\downarrow} \neq 0$ if the condition $\delta n = 0$ is imposed before applying the stationarity condition (case described in [4, 5, 8]). In other terms, introducing spin-flip scattering in the model does not help in figuring out what definition of the stationary state is the more relevant.

V. TRANSPORT EQUATIONS

In this Section, the transport equations Eq.(13) in the presence of the chemical potential Eq.(14) are analyzed. We show in Subsection **A** that the transport equation is equivalent to the Dyakonov-Perel equations in the case of the electric field $E_{y\uparrow} = E_y$ is spin-independent. In Subsection **B**, the transport equation Eq.(13) are analyzed in terms of a generalized Dyakonov-Perel equation.

A. Case of spin-independent electric field \vec{E}

Introducing the chemical potential $\mu_{\uparrow} = kT \ln(n_{\uparrow}) + V + \mu_{0\uparrow}$ in Eq.(13), we obtain for $\vec{\nabla} \mu_{0\uparrow} = 0$:

$$\vec{J}_{\uparrow} = q\eta n_{\uparrow} \vec{E} - D \vec{\nabla} n_{\uparrow} \pm \vec{p} \times \left(-qn_{\uparrow} \eta_{so} \vec{E} + D_{so} \vec{\nabla} n_{\uparrow} \right) \quad (27)$$

where $D_{so} = \eta_{so} kT$ is the spin-orbit diffusion constant.

Defining the asymmetry of charge carriers density between the two spin channels $\Delta n = n_{\uparrow} - n_{\downarrow}$ and $n = n_{\uparrow} + n_{\downarrow}$, we can define the charge current $\vec{J}_c = \vec{J}_{\uparrow} + \vec{J}_{\downarrow}$ and the spin current $\vec{J}_s = \vec{J}_{\uparrow} - \vec{J}_{\downarrow}$ by summing and subtracting the two Eqs.(27):

$$\begin{aligned} \vec{J}_c &= q\eta n \vec{E} - D \vec{\nabla} n + \vec{p} \times \left(-q\eta_{so} \Delta n \vec{E} + D_{so} \vec{\nabla} (\Delta n) \right) \\ \vec{J}_s &= q\eta \Delta n \vec{E} - D \vec{\nabla} (\Delta n) + \vec{p} \times \left(-q\eta_{so} n \vec{E} + D_{so} \vec{\nabla} n \right) \end{aligned} \quad (28)$$

where the unit vector \vec{p} defines the orientation of the spin-polarization. We can check that Eq.(28) is equivalent to the Dyakonov-Perel equations (in the Dyakonov-Perel form [1, 2]),

when we introduce the spin-polarisation \vec{P} :

$$\begin{aligned} \vec{J}_c &= \tilde{\mu} n \vec{E} + \tilde{D} \vec{\nabla} n + b \vec{E} \times \vec{P} + \delta \vec{r} \otimes P \\ q_{ij} &= -\tilde{\mu} E_i P_j - \tilde{D} \frac{\partial P_j}{\partial x_i} + \epsilon_{ijk} \left(b n E_k + \delta \frac{\partial n}{\partial x_k} \right). \end{aligned} \quad (29)$$

The second tensorial equation in Eqs.(29) can equivalently be put into the vector form:

$$\vec{q}_j \equiv \vec{J}_s = -\tilde{\mu} \vec{E} P_j - \tilde{D} \vec{\nabla} P_j + \vec{e}_{x_j} \times b n \vec{E} + \delta \vec{r} \otimes (n \vec{e}_{x_j}) \quad (30)$$

where j corresponds to the projection along the \vec{p} direction ($\vec{e}_{x_j} = \vec{p}$). Since we have $\vec{r} \otimes (\Delta n \vec{p}) = -\vec{p} \times \vec{\nabla}(\Delta n)$, the Dyakonov model defined by Eqs.(29) is equivalent to the two channel model providing the phenomenological constants of the model are defined as follows: the mobility of charge carriers is $\tilde{\mu} = q\eta$, the spin-orbit mobility is $b = -q\eta_{so}$, the diffusion constant is $\tilde{D} = -D$, the spin-orbit diffusion constant is $\delta = D_{so}$, and the spin-polarization is $\vec{P} = -\Delta n \vec{p}$.

The main consequence of Eq.(28) can be calculated in the following simple configuration in the bulk. Since the spin-Hall effect is characterized by zero charge accumulation $\delta n = 0$, the electric field along the axis y also vanishes, $E_y = 0$, in the absence of other contributions. For the bulk approximation the diffusion terms and the spin accumulation Δn vanish, and the DP equations reduces to $J_{xc} = q\eta n E_x^0$ and $J_{sy} = q\eta_{so} n E_x^0$ (and $J_{cx} = J_{sx} = 0$). **These equations necessarily lead to a non-vanishing pure spin-current** $J_{y\uparrow} = -J_{y\downarrow} \neq 0$. Here, the spin-Hall angle is defined by the ratio of the spin currents J_{sy} to the injected current J_{cx} : $\theta_{SH} = J_{sy}/J_{cx} = \eta_{so}/\eta$. The form Eq.(28) of the DP equations is probably the main motivation for the usual description of the SHE.

B. Case of spin-dependent electric field $\vec{E}_{\uparrow\downarrow}$

We discuss now the case developed in this work, in which the electric field $E_{y\uparrow}$ is due to Gauss's law $\partial E_{y\uparrow}/\partial y = q\delta n_{\uparrow}/\epsilon$ and is spin-dependent. Introducing the chemical potential Eq.(14) into the transport equations Eq.(13) leads to the following generalized DP equations:

$$\vec{J}_{\uparrow\downarrow} = q\eta n_{\uparrow\downarrow} \vec{E}_{\uparrow\downarrow} - D \vec{\nabla} n_{\uparrow\downarrow} \pm \vec{p} \times \left(-q n_{\uparrow\downarrow} \eta_{so} \vec{E}_{\uparrow\downarrow} + D_{so} \vec{\nabla} n_{\uparrow\downarrow} \right) \quad (31)$$

or:

$$\begin{aligned} \vec{J}_c &= q\eta \left(n_{\uparrow} \vec{E}_{\uparrow} + n_{\downarrow} \vec{E}_{\downarrow} \right) - D \vec{\nabla} n + \vec{p} \times \left(-q\eta_{so} \left(n_{\uparrow} \vec{E}_{\uparrow} - n_{\downarrow} \vec{E}_{\downarrow} \right) + D_{so} \vec{\nabla}(\Delta n) \right) \\ \vec{J}_s &= q\eta \left(n_{\uparrow} \vec{E}_{\uparrow} - n_{\downarrow} \vec{E}_{\downarrow} \right) - D \vec{\nabla}(\Delta n) + \vec{p} \times \left(-q\eta_{so} \left(n_{\uparrow} \vec{E}_{\uparrow} + n_{\downarrow} \vec{E}_{\downarrow} \right) + D_{so} \vec{\nabla} n \right) \end{aligned} \quad (32)$$

The particular case where $\vec{E}_{\uparrow} = \vec{E}$ leads to the DP equations obtained in the previous Subsection **A**.

Let us look at the simple situation calculated in the previous subsection. Since the spin-Hall effect is characterized by the symmetry $\delta n_{\uparrow} = -\delta n_{\downarrow}$ (when η is spin-independent), the relation $E_{y\uparrow} = -E_{y\downarrow}$ is verified when other contributions are absent. We then have $\Delta E = E_{y\uparrow} - E_{y\downarrow} \neq 0$ and $E_y = E_{y\uparrow} + E_{y\downarrow} = 0$. For the bulk approximation the diffusion terms and the spin-accumulation Δn vanish, such that the generalized DP equations Eqs.(32) are reduced to $J_{cx} = q\eta n E_x^0 + q\eta_{so} n E_{y\uparrow}$ and $J_{sy} = q\eta n E_{y\uparrow} - q\eta_{so} n E_x^0$ ($J_{cy} = J_{sx} = 0$).

The stationarity condition $J_{sy} = J_{sx} = 0$ **generates the spin-dependent electric fields**: $E_{y\uparrow} = \mp \theta_{SH} E_x^0$. From the experimental point of view, the two results are very similar in both cases **A** and **B**, except that a **pure spin-voltage** $V_{y\uparrow}$ is measured instead of a pure spin-current $J_{y\uparrow}$, and the **spin-Hall angle** $\theta_{SH} \equiv \eta_{so}/\eta$ is now measured as the ratio of the electric fields instead of the ratio of the electric currents.

VI. CONCLUSION

We have analysed the spin-Hall effect in the framework of the two spin-channel model (labeled by the indices $\uparrow\downarrow$) depicted in Fig.1. In this context, the spin-Hall effect and the corresponding spin-accumulation at the edges is a consequence of the electrostatic charge accumulation that is generated in each spin-channel δn_{\uparrow} by the spin-orbit effective magnetic field $\vec{H}_{so\uparrow}$ over the screening distance λ_D . The main property of the spin-orbit coupling is expressed by the reciprocity relation $H_{so\uparrow} = \pm H_{so}$ which leads to inverse charge accumulation $\delta n_{\uparrow} = -\delta n_{\downarrow}$ at the same edge.

It is shown that two possible types of stationary states can be applied to this SHE model. On one hand, the stationary condition can be applied before summing over the two spin channels, such that the stationary state is defined by zero electric charge accumulation, non-zero spin-accumulation at the edges, and zero transverse current $J_{y\uparrow} = 0$ inside the bulk. On the other hand, the stationary condition can be applied after summing over the two spin channels, so that the stationary state is defined by zero electric charge accumulation, non-zero spin-accumulation, and non-zero pure spin-current $J_{y\uparrow} = -J_{y\downarrow}$ inside the bulk. The first situation corresponds to a generation of an electric field in each channel $E_{y\uparrow}$ such that $\partial E_{y\uparrow}/\partial y = q\delta n_{\uparrow}/\epsilon$ and a total electric field E_y such that $\partial E_y/\partial y = 0$.

It is shown that the transport equations that corresponds to the last situation (with the generation of pure spin-current in the bulk) are the Dyakonov-Perel equations, in which the electric field \vec{E} is spin-independent. The former situation (without zero transverse current in the bulk) corresponds to a generalized Dyakonov-Perel equation, in which the electric field is spin-dependent.

Finally, it is clear that the stationary state with the pure spin-current dissipates more than that with zero transverse current. From that point of view, the corresponding system is more constrained than that without pure spin-current.

From the experimental point of view, the presence or absence of the charge accumulation δn_{\uparrow} or spin-dependent voltage V_{\uparrow} in each spin-channel is probably impossible to measure (only the spin-accumulation and total voltage is accessible). However, discriminating the stationary states with or without transverse currents $J_{y\uparrow}$ in the bulk should in principle be easy. Indeed, the ratio of the resistances measured with or without the pure spin-current is given by the square of the spin-Hall angle θ_{SH} , as discussed in reference [10].

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